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Supporting Information

for

Mechanical property measurements enabled by short-term Fourier-transform of atomic force microscopy thermal deflection analysis

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Equations of motion of the cantilever dynamics models and additional experimental data

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Equations of Dispersion Relations for Three Cantilever Models

Solution to model 1 dispersion

$$\sinh(k_n L)\cos(k_n L) - \sin(k_n L)\cosh(k_n L) = \frac{(k_n L)^3 k_c}{3k^*}(1 + \cos(k_n L)\cosh(k_n L)) \tag{S1}$$

Solution to model 2 dispersion

$$\begin{aligned} &-(\cosh(k_n L_1)\sin(k_n L_1) - \sinh(k_n L_1)\cos(k_n L_1))(1 + \cos(k_n L')\cosh(k_n L')) \\ &-(\cosh(k_n L')\sin(k_n L') - \sinh(k_n L')\cos(k_n L'))(1 - \cos(k_n L_1)\cosh(k_n L_1)) \\ &\qquad = 2k_n^3 \frac{EI}{k^*}[1 + \cos(k_n(L_1 + L'))\cosh(k_n(L_1 + L'))] \end{aligned} \tag{S2}$$

Solution to model 3 dispersion

$$\frac{k^*}{k_c} = \frac{-B \pm \sqrt{B^2 - 4AC}}{6A} \tag{S3}$$

$$A = \left(\frac{\kappa}{k^*}\right)\left(\frac{h}{L_1}\right)^2 (1 - \cos(x)\cosh(xL_1))(1 + \cos(xL')\cosh(xL')) \tag{S4}$$

$$B = B_1 + B_2 + B_3 \tag{S5}$$

$$C = 2(xL_1)^4(1 + \cos(xL_1)\cosh(xL_1)) \tag{S6}$$

$$\begin{aligned} B_1 = &\left(\frac{h}{L_1}\right)^2 (xL_1)^3 \left(\sin^2(\alpha) + \frac{\kappa}{k_c}\cos^2(\alpha)\right) \\ &[(1 + \cos(xL')\cosh(xL'))(\sin(xL_1)\cosh(nL_1) + \cos(xL_1)\sinh(xL_1)) \\ &\qquad - (1 - \cos(nL_1)\cosh(nL_1))(\sin(xL')\cosh(xL') + \cos(xL')\sinh(xL'))] \end{aligned} \tag{S7}$$

$$B_2 = 2\left(\frac{h}{L_1}\right)(xL_1)^2\left(\frac{\kappa}{k_\mathrm{c}}\cos(\alpha)\sin(\alpha)\right)$$
$$[(1+\cos(xL')\cosh(xL'))(\sin(xL_1)\sinh(nL_1))$$
$$+(1-\cos(nL_1)\cosh(nL_1))(\sin(xL')\sinh(xL'))] \quad \text{(S8)}$$

$$B_3 = (xL_1)(\cos^2(\alpha)+\frac{\kappa}{k_\mathrm{c}}\sin^2(\alpha))$$
$$[(1+\cos(xL')\cosh(xL'))(\sin(xL_1)\cosh(nL_1)-\cos(xL_1)\sinh(xL_1))$$
$$-(1-\cos(nL_1)\cosh(nL_1))(\sin(xL')\cosh(xL')-\cos(xL')\sinh(xL'))] \quad \text{(S9)}$$

$$G^* = \left(\frac{2-\nu_1}{G_1}+\frac{2-\nu_2}{G_2}\right)^{-1} \quad \text{(S10)}$$

$$G = \frac{1}{2}\left(\frac{E}{1+\nu}\right) \quad \text{(S11)}$$

$$\kappa = 8G^*a = \frac{8G^*k^*}{2E^*} = \frac{4k^*\left(\frac{2-\nu_1}{\frac{1}{2}\frac{E_1}{1+\nu_1}}\right)+\left(\frac{2-\nu_1}{\frac{1}{2}\frac{E_1}{1+\nu_1}}\right)}{E^*} \quad \text{(S12)}$$

Cantilever Frequency Power Spectrum

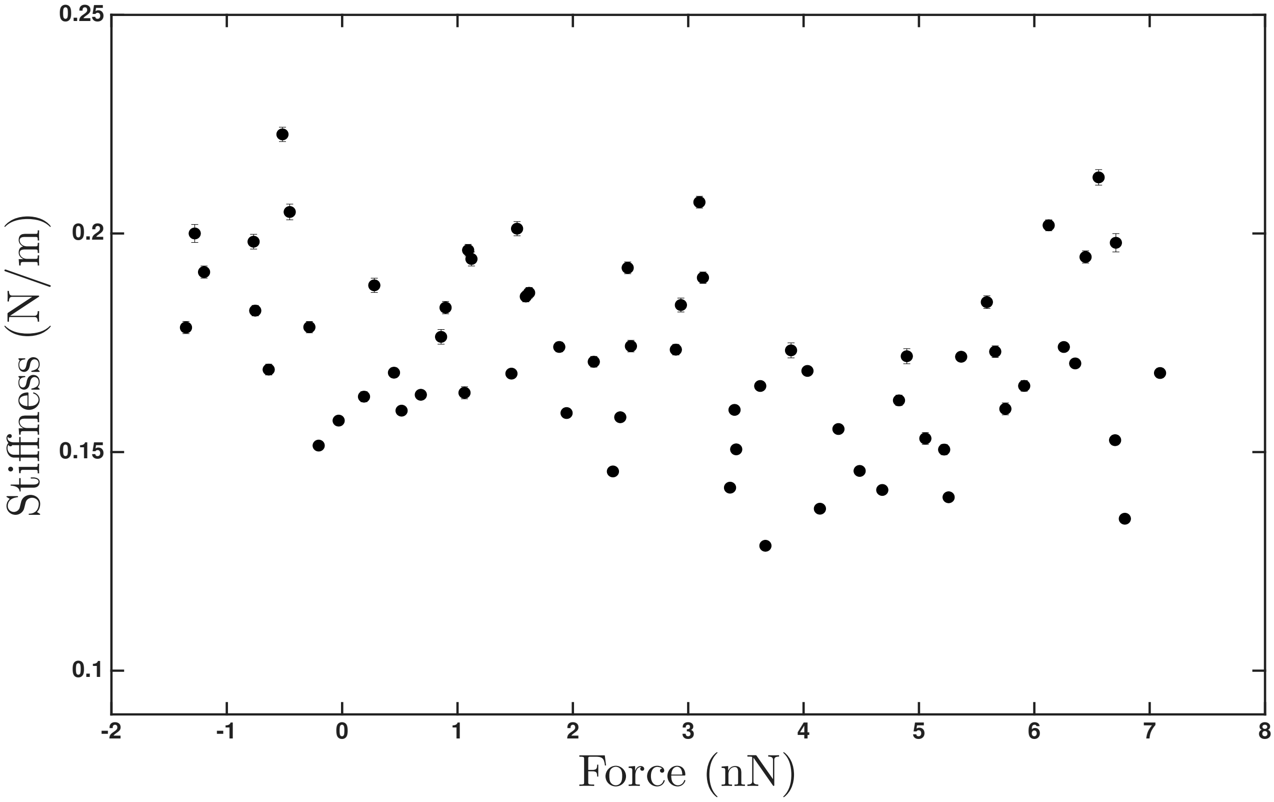

Figure S1: Stiffness versus normal force determined from fits of the first normal resonant mode peak in the power spectra of the contact portion of a force spectroscopy sweep.

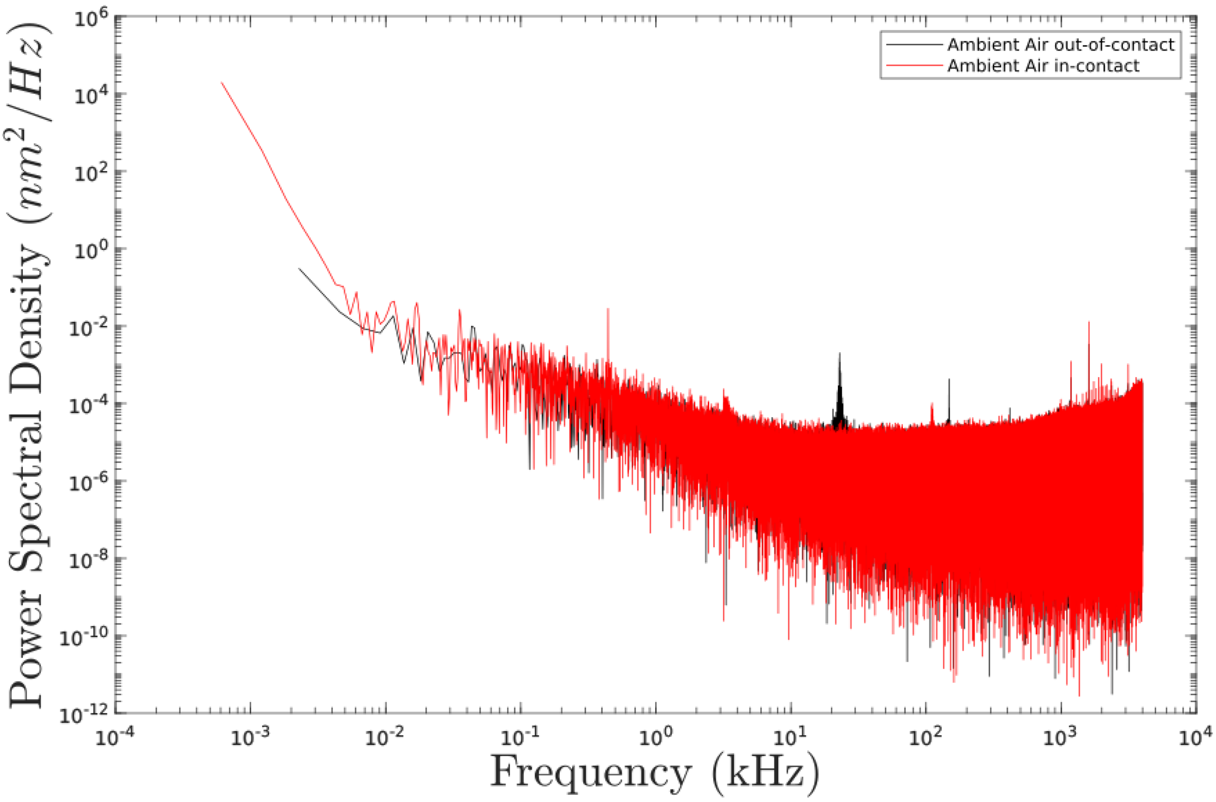

Figure S2: Fourier transform of the out-of-contact portion of a diamond coated probe having a spring constant between 20-40 N·m^{-1} on a silicon substrate. The out of contact portion is shown in black and the in-contact portion in red, highlighting the change in the resonant peak locations and shapes between these two stages of the measurement.

Elastic Modulus Determination of PEO

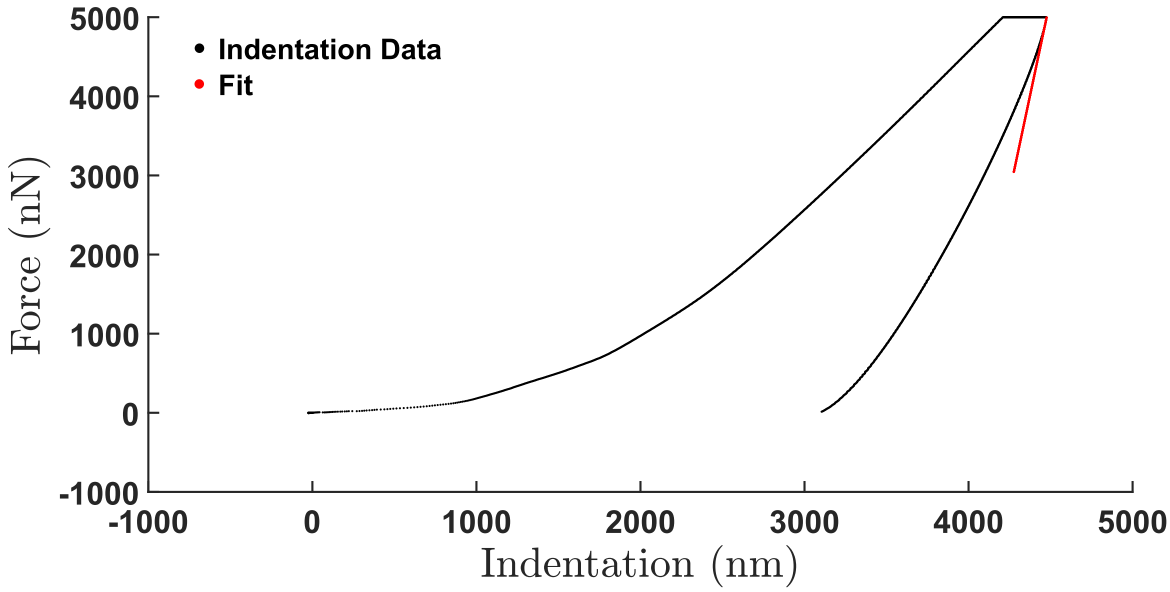

Figure S3: Example elastic unloading curves for the PEO sample obtained from nanoindentation experiments with a Berkovich indenter. Slope of curve in red is the linear fit used to determine the Young's modulus of the sample.