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1 **Spin supercurrent injection by magnetization precession**

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7 **Abstract**

8 In this work we theoretically investigate the spin dynamics of superconducting condensate prox-
9 imity coupled with the single FMR mode uniform periodical precessing magnetization. The the-
10 oretical model of the inverse proximity effect is built within quasiclassical formalism using time-
11 dependent Usadel equations. Frequency representation of Usadel equations allows to turn non-
12 stationary periodic problem to the stationary one. We numerically solve the derived frequency-
13 dependent Usadel equations and calculate non-stationary distributions of the spin supercurrent and
14 induced magnetization inside the superconductor/ferromagnetic insulator hybrid structure.

15 **Keywords**

16 superconducting spintronics; proximity effect; ferromagnetic resonance

17 **Introduction**

18 Spin flow creation and manipulation in superconducting hybrid systems became a very active re-
19 search area during the last decade because of possibility of creation spin supercurrents with much
20 larger relaxation lengths and spin lifetimes [1]. Creation of persistent spin currents in superconduc-
21 tors opens new ways for the development of prospective spintronic devices like magnon transistors
22 [2,3] and superconducting magnon crystals [4]. In this context, the challenge of the superconduc-

23 tor spin injection is one of the central problems in modern superconducting spintronics. There are
24 several ways of spin current injection into the superconductor, for example spin Hall effect [5] ,
25 spin Seebeck [6] effect, ferromagnetic resonance spin pumping [7,8]. The spin pumping technique
26 in hybrid structure, consisting of ferromagnetic insulator and superconductor, is considered to be
27 the most preferable way to inject spin current, due to the absence of Joule heating. Moreover, prox-
28 imity coupling between magnetic excitations plays a crucial role in ferromagnetic Josephson junc-
29 tions [9-12] and mesoscopic structures [13]. The latest experimental researches [5,8,14] show that
30 the interaction between the superconducting correlations and spin waves, influences both dynam-
31 ics of superconducting and magnetic films. Interfacial exchange interaction between cooper pairs
32 and magnons results in non-stationary induced magnetization and spin currents in superconducting
33 film and changes the magnetic excitations spectrum inside the ferromagnetic insulator [15]. De-
34 spite the large number of discussions in experimental works, there is no clear understanding of the
35 interplay between superconducting and magnetic excitations inside proximity coupled hybrid struc-
36 tures. That is why developing a consistent theory of the inverse proximity effect is one of the cen-
37 tral topics of modern superconducting spintronics. There is a series of theoretical papers [7,16-19]
38 describing the spin current injection and induced magnetization generation in microscopic [7,16]
39 and quasiclassical [17-19] frameworks. However, the main subject of this works is the magnetic
40 excitation spectrum in hybrid structures. Most of the works ignore the dynamics of non-uniform
41 distributions of induced magnetization and spin current inside the superconducting film, which
42 can be called the inverse proximity effect. Distributions of spin-current and induced magnetization
43 were calculated in recent works [20,21], where the authors investigate spin current flow through the
44 Josephson-like trilayer structures.

45 The quasiclassical theory of proximity effect in superconductor/ferromagnetic insulator hybrid
46 structures was applied to describe non-uniform phenomena, like generation of spin transfer torques,
47 non-uniform thermoelectric effects, domain wall movement, etc. The theoretical description of
48 the dynamic proximity effect is the more complex task because of the double time structure of
49 non-stationary Usadel equation. The recent successes in quasiclassical boundary conditions the-

50 ory [22,23] make it possible to develop adequate models of the proximity effect in the different
 51 types of superconducting hybrid structures. Quasicalssical boundary conditions can successfully
 52 describe the interfaces between the superconductor and weak or strong ferromagnets [22-24], nor-
 53 mal metals [25-27], half-metals [28], etc. The first attempts to implement non-stationary, adiabatic,
 54 quasicalssical boundary conditions were done in the works [18,19]. In this work, we develop a the-
 55 ory based on Usadel equations, combined with the adiabatic, non-stationary boundary conditions.
 56 We show that adiabatic approximation is useful in the wide range of magnetization precession fre-
 57 quencies. The main goal of our theory is to describe the dynamics of the spin current and induced
 58 magnetization inside the superconducting film, which is in contact with the ferromagnetic insulator
 59 layer. We calculate spin current and induced magnetization, not only at the interface of the hybrid
 60 structure, but also inside the superconducting metal film.

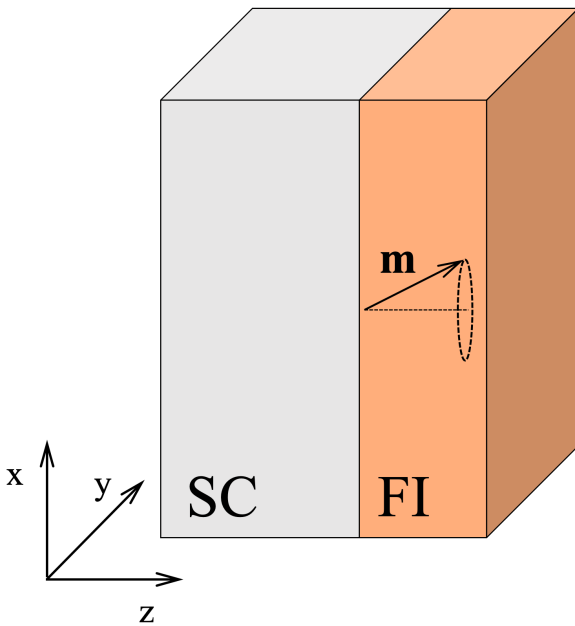


Figure 1: Investigated hybrid superconducting structure: ferromagnetic insulator(FI) adjacent to superconductor (SC). Magnetization \mathbf{m} in ferromagnetic insulator layer is uniform and its direction precesses with the cyclic frequency Ω , z coordinate of the interface is equal to the superconductive layer thickness.

61 Model

62 The structure is schematically depicted in Figure 1. The spin current is injected from the ferromagnetic insulator (FI) to the superconducting film (SC). The thickness of the ferromagnetic insulator doesn't matter because the superconducting correlations do not penetrate inside the insulating material. Uniform magnetization periodically precesses inside the ferromagnetic insulator with a cyclic frequency Ω . To describe the non-stationary state of the superconducting condensate we use the formalism of two-time quasiclassical Green functions in Nambu-Spin-Keldysh space[28]. We expand the Green function assuming the weak proximity effect [28] with the ferromagnetic insulator: $\check{g}(t_1, t_2) \approx \check{g}^{(0)} + \check{g}^{(1)}$. To handle the expansion of the order parameter correctly, we should cancel the odd orders of the perturbation, because the triplet Green function components do not contribute to the order parameter. Only even orders of the perturbation series determine its correction. Thus, the superconducting order parameter in the linear regime, has only a zero-order term in expansion. The resulting dynamics of the superconducting condensate in weak proximity effect regime, can be described via the non-stationary Usadel equation [18,19,29]:

$$65 \quad \hbar\{\check{\rho}_4\partial_t, \check{g}\}_t - \hbar D\partial_z(\check{g} \circ \partial_z\check{g}) = i\left[\check{\Delta}^{(0)}, \check{g}\right], \quad (1)$$

66 where $\check{\Delta}^{(0)}$ - the stationary, BCS superconducting order parameter matrix [28], D - the diffusion constant, $\check{\rho}_4 = \sigma_0 \otimes \sigma_x \otimes i\sigma_y$ -the auxiliary matrix in Nambu-spin-Keldysh space, \circ - the time convolution operator, the anticommutator $\{f, g\}_t = f(t_1)g(t_1, t_2) + g(t_1, t_2)f(t_2)$. We have dropped the coordinate dependence of Green functions and of the order parameter for the simplicity of notation. We consider time-dependent magnetization at the interface as a adiabatic perturbation which changes slowly compared to the timescale of the superconducting system : $\hbar\Omega \ll \Delta$.

67 In general, the equation can be solved numerically within the mixed representation formalism. The time-periodicity condition allows us to represent spin-current and induced magnetization as time-harmonic variables:

$$68 \quad \mathbf{j}_z^s(z, t) = \mathbf{j}_z^s(z) e^{i\Omega t} \quad (2)$$

$$\mathbf{M}(z, t) = \mathbf{M}(z) e^{i\Omega t} \quad (3)$$

where $t = (t_1 + t_2)/2$ is the centre-of-mass time argument. To form a closed set of equations we should add the equation for the normalization condition [29] in mixed representation.

Results and Discussion

For the numerical calculations, we have considered niobium as a superconducting metal with the following parameters $T_c = 9.2$ K, $\Delta^{(0)} \approx 1.76k_B \cdot T_c = 14$ meV, $D = 0.8 \cdot 10^{-3}$ m²/s, $\epsilon_F \approx 5.32$ eV, we approximate DOS on the Fermi level with the free electron gas value $N_0 \approx 4.9 \cdot 10^{46} J^{-1} m^{-3}$, the coherence length has been estimated using $\xi_0 = \sqrt{\hbar D / 2\pi k_B T_c}$ where k_B is the Boltzmann constant, $\xi_0 \approx 1.1 \cdot 10^{-8}$ m. We numerically solve the mixed representation of the equation (1) with the normalization condition. To obtain physical observables from the quasiclassical, Green functions, we should find the harmonic coefficients (2),(3) and directly calculate observable values at the space-time points. In this work, we are interested in the calculation of spin currents and induced magnetization distributions along the thickness of the superconducting film. The dynamics of any observable will be periodic and can be characterized by its amplitude value. Thus, we only need to calculate the doubled, absolute value of the coefficients (2) and (3), which are exactly the amplitudes of the spin current and magnetization in the linear regime. Now let us consider the doubled Fourier coefficients for the induced magnetization. One can see that the curves for the different precession frequencies coincide. This happens because the absolute value of the projection of the magnetization vector at the interface does not change with the changing of the precession frequency. However, a more complicated picture is anticipated, if we take into account the non-adiabatic process at the interface. At first glance, there is no possibility of the non-adiabatic processes, because the ratio $\Delta/\hbar\Omega \sim 50$ for the Nb / YIG hybrid structure. But the superconducting order parameter can be dramatically reduced at the interface between the superconductor and ferromagnetic due to the strong proximity effect. If the order parameter at the interface is suppressed by the proximity effect, this ratio can move close enough to 1 and non-adiabatic effects will come into play. These complex dynamics can affect the quasiparticle generation at the interface. and also

112 cause significant suppression of the superconductivity. We do not explore the stationary compo-
 113 nent of the induced magnetization because it was done in [30]. In this work, we are interested in
 114 the dynamic components of the induced magnetization and spin current. Both spin current and
 115 induced magnetization in the superconductor are originated from the singlet-triplet Cooper pairs
 116 conversion mechanism. The spin current can be induced only by the non-stationary flow of triplet
 117 Cooper pairs, just as in the conventional spin-pumping bilayer structure with normal metal [31].
 118 Thus, spin currents cannot emerge when the magnetization is stationary inside the ferromagnetic
 119 insulator layer. However, there is a possibility to induce stationary pure spin currents inside trilayer
 120 superconducting structures [1].

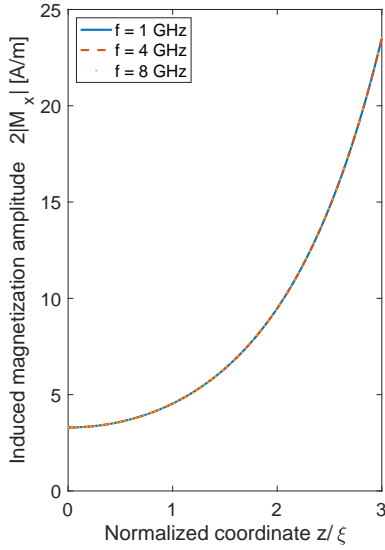


Figure 2: Distribution of the induced magnetization inside the superconducting layer at different frequencies of the magnetization precession

121 Figure 2 displays the exponential-like decay of the induced magnetization inside the supercon-
 122 ducting film. The induced magnetization is created by the triplet superconducting correlations,

123 whose concentration reaches the maximum value at the interface due to the singlet-triplet conver-
 124 sion process. Suppression of superconductivity at the interface can give rise to some interesting,
 125 non-adiabatic spin dynamics. In Figure 3, one can see the amplitude of the induced magnetization
 126 at the interface between the superconductor and ferromagnetic insulator.

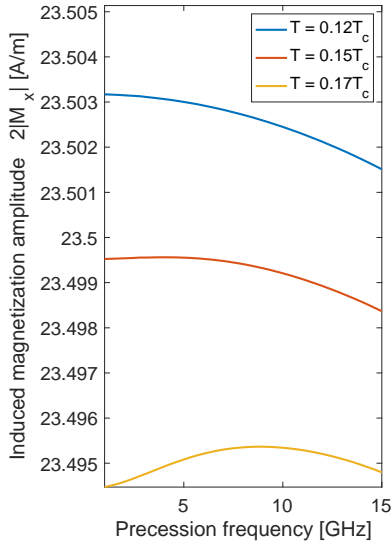


Figure 3: Induced magnetization at the interface between the superconductor and ferromagnetic insulator

127 As one can see from Figure 3, the induced magnetization distribution depends non-monotonically
 128 on the precession frequency. Moreover, the peak becomes more visible with the increasing of the
 129 temperature, even if we do not take into account the thermal suppression of the superconducting
 130 order parameter. We predict that the competition between two different spin pumping mechanisms
 131 can explain this interesting behaviour. The first mechanism is the adiabatic spin pumping of the
 132 superconducting condensate and the second one is the spin pumping of the thermally-generated
 133 quasiparticles e. g. normal electrons and holes. We should remember that increasing the excita-

134 tion frequency can cause the suppression or destruction of superconductivity at the interface. This
 135 means that the induced magnetization of the triplet correlations will also decrease. However, the
 136 spin pumping in normal metals goes exactly in the opposite way: the spin density is growing with
 137 the increasing of the excitation frequency. According to different spin pumping mechanisms, the
 138 induced magnetization is the sum of the quasiparticle spin density and triplet Cooper-pairs mag-
 139 netization. In general, the interplay between the magnetization precession and proximity effect
 140 can completely destroy superconductivity at the interface. During the interaction with the time-
 141 dependent magnetization at the interface, electrons from the superconductor can gain or lose addi-
 142 tional magnon energy quanta $\hbar\Omega$. If $\hbar\Omega \sim \Delta$ at the interface, the inelastic electron scattering causes
 143 the breaking of the Cooper pairs and generates quasiparticles above and below the Fermi surface.
 144 That is why the non-adiabatic effects can potentially play extremely important role in the spin dy-
 145 namics of the superconductor/ferromagnetic hybrid structures.

146 Next, we consider the distribution of the spin current. The amplitude of spin current is normal-
 147 ized by the factor $j_{s0} = (\hbar/2e) j_{e0}$. The charge current density normalization factor is $j_{e0} =$
 148 $2eN_0D\Delta^{(0)}/\xi = 6.26210^6 A/cm^2$. The spin current distribution amplitudes are depicted in Fig-
 149 ure 4.

150 One can see that the spin current amplitudes decay similarly to the induced magnetization. How-
 151 ever, the amplitude of the spin current strongly depends on the frequency of the magnetiza-
 152 tion precession. This effect is similar to ferromagnetic resonance spin pumping in the normal
 153 metal/ferromagnetic insulator structures. In the case of spin pumping into normal metal, the de-
 154 cay of the spin current is a consequence of the spin relaxation processes, but we do not take into
 155 account any of the spin relaxation mechanisms within our model. Therefore, we conclude that the
 156 main mechanism of the spin current reduction is similar to that for the induced magnetization: the
 157 decay of the spin current density corresponds to the lowering of the triplet pairs density away from
 158 the magnetic interface, because of singlet-triplet conversion weakening.

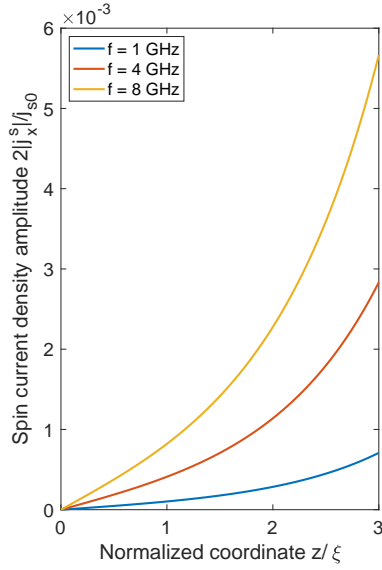


Figure 4: The distribution of the spin current density inside the superconducting layer at the different frequencies of the magnetization precession

159 Conclusion

160 In this work, we have investigated the simplest case of the linear adiabatic dynamics caused by the
 161 proximity of the superconductor with the ferromagnetic insulator. It was found that the spin cur-
 162 rent density amplitude is proportional to the frequency of the magnetization precession. Distribu-
 163 tions of the induced magnetization and spin supercurrent are similar to those of the spin pumping
 164 in normal metal / ferromagnetic insulator hybrid structures. But the spin current and spin density
 165 penetrate inside superconducting film on the distances much longer than in normal metals. This
 166 behaviour is a result of the adiabatic singlet-triplet cooper pair conversion process at the interface.
 167 However, we have pointed out that the spin current generation at the interface can be largely af-
 168 fected by the non-adiabatic effects, which can arise due to the significant suppression of the super-
 169 conducting order parameter near the ferromagnetic insulator. That is why the non-perturbative de-

170 scription of the dynamic inverse proximity effect must include both adiabatical and non-adiabatical
171 processes. In other words, we should take into account the inelastic electron scattering at the inter-
172 face. The time-dependent inelastic scattering leads to the breaking of the superconducting corre-
173 lations at the interface and the injection of the quasiparticles inside the superconducting film. The
174 results reported in this work, demonstrate the rich potential of the dynamic inverse proximity effect
175 in hybrid superconductor / ferromagnetic insulator structures, making them promising candidates
176 for novel spintronic devices.

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